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APPLICATION OF THE DETERMINANT APPARATUS IN MODELING THE WEAR KINETICS AND MECHANICAL STRENGTH OF MACHINE PARTS.

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Abstract: This paper delineates a methodology for evaluating the tribological wear kinetics and mechanical strength of machine components under stress states using the linear algebra determinant apparatus. Utilizing agricultural cultivator sweeps operating in intense abrasive environments as an experimental baseline, a multi-parametric model is developed to map wear depth against contact pressure, sliding velocity, and operational duration. The structural stability of the predictive model is verified through system determinant evaluation, while individual tribological sensitivity coefficients are isolated via Cramer's matrix routine. Furthermore, structural integrity is monitored by processing the characteristic determinant of the stress tensor to analytically extract principal stresses. Experimental validation demonstrates a high degree of correlation with physical wear measurements, proving the efficacy of matrix-based algorithms for automated service-life forecasting.

Keywords: Determinant apparatus, wear kinetics, mechanical strength, linear algebra, Cramer's rule, stress tensor, tribomechanical model, operational resource forecasting.

Annotatsiya: Maqolada mashina detallarining yeyilish kinetikasi va mexanik mustahkamligini baholashda chiziqli algebra, xususan, determinantlar apparatidan foydalanish uslubiyoti yoritilgan. Tajriba obyekti sifatida abraziv muhitda ishlovchi chizel-kultivator pichoqlari olingan bo‘lib, ularning yeyilishi solishtirma bosim, tezlik va vaqt omillariga bog‘liq holda matritsali model ko‘rinishida ifodalangan. Asosiy determinant orqali modelning turg‘unligi asoslangan va Kramer qoidasi yordamida empirik koeffitsiyentlar aniqlangan. Shuningdek, kuchlanishlar tenzori determinanti orqali bosh kuchlanishlarni hisoblash va detal resursini prognozlash algoritmi ishlab chiqilgan. Modelning eksperimental tekshiruvi natijasida nisbiy xatolik minimal darajani tashkil etib, muhandislik hisob-kitoblari uchun yuqori aniqlikni ko‘rsatdi.

Kalit so‘zlar: Determinantlar apparati, yeyilish kinetikasi, mexanik mustahkamlik, chiziqli algebra, Kramer qoidasi, kuchlanishlar tenzori, tribologik model, ekspluatatsiya resursi.

Аннотация: В статье рассматривается методология применения аппарата детерминантов линейной алгебры для оценки кинетики трибологического износа и механической прочности деталей машин. В качестве объекта исследования приняты рабочие органы чизель-культиватора, работающие в абразивной среде. Линейный износ деталей представлен в виде матричной модели, зависящей от удельного давления, скорости скольжения и времени эксплуатации. Устойчивость модели обоснована через вычисление главного детерминанта системы, а эмпирические коэффициенты чувствительности определены по правилу Крамера. Также предложен алгоритм прогнозирования остаточного ресурса деталей с помощью характеристического детерминанта тензора напряжений. Экспериментальная проверка разработанной модели показала высокую точность метода, что подтверждает его эффективность для инженерных расчетов.

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Ключевые слова: Аппарат детерминантов, кинетика износа, механическая прочность, линейная алгебра, правило Крамера, тензор напряжений, трибомеханическая модель, прогнозирование ресурса.

1. INTRODUCTION.

The operational reliability and structural integrity of modern machinery depend heavily on the degradation mechanics of their conjugating components. In mechanical engineering and agricultural machinery design, predicting the degradation of contact surfaces subjected to intense abrasive environments, cyclic dynamic loads, and thermal fluctuations remains a fundamental challenge. Elements such as cultivator blades, transmission shafts, and heavy-duty industrial bearings continuously undergo progressive material detachment, structurally classified as wear.

Surface wear is a highly non-linear, multi-factorial tribological phenomenon governed by a complex interplay of operational velocity (v), contact pressure (P), structural temperature, micro-hardness of the substrate material, and cumulative operational time (t). Traditional empirical approaches, such as standard regression analysis, often treat these variables in isolation or fail to mathematically address the systemic stability of the coupled equations under volatile physical boundaries. When analyzing multi-variable tribological systems, formulating exact analytical solutions requires mathematical frameworks capable of assessing both the existence of unique solutions and the structural stability of the underlying mathematical models.

To bridge this gap, advanced linear algebra tools—specifically the determinant apparatus—offer a computationally elegant and rigorous framework. By structuring multi-variable tribological and stress equations into square matrices, determinants enable the direct execution of Cramer's rule to isolate independent empirical coefficients. Beyond solving static linear systems, the evaluation of the system's primary determinant ($\Delta \neq 0$) serves as a fundamental mathematical indicator of model stability and non-singularity. In dynamic mechanics, Wronskian and Jacobian determinants are instrumental in testing the linear independence of degradation functions and defining boundary transitions where stable linear wear shifts into catastrophic, accelerating failure zones.

Furthermore, analyzing mechanical strength requires mapping multi-axial stress states to evaluate structural failure limits. Evaluating the characteristic determinant of the stress tensor allows for the analytical extraction of principal stresses (σ_1, σ_2, σ_3,), eliminating reliance on iterative approximations or computationally heavy purely-numerical simulations.

Consequently, integrating the determinant apparatus into tribomechanical modeling enhances mathematical precision, reduces the structural margins of error, and minimizes computational processing time in automated prediction algorithms.

The primary objective of this study is to develop a mathematically robust, multi-parametric linear model utilizing the determinant apparatus to precisely chart the wear kinetics and assess the mechanical strength parameters of critical machine parts. By validating this matrix-based methodology against concrete experimental data derived from abrasive wear testing of agricultural components, this paper establishes a highly reproducible analytical framework for service-life optimization and operational resource forecasting in structural engineering applications.

2. MATERIALS AND METHODS.

To analytically model the multi-parametric nature of wear kinetics and evaluate mechanical strength boundaries, this study establishes a mathematical framework anchored in matrix algebra and the determinant apparatus. The degradation of contact surfaces under abrasive wear is fundamentally governed by operational pressure, relative sliding velocity, and exposure duration. To map these

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interactions into a deterministic mathematical model, the differential form of Archard's wear law is generalized into a multi-variable linear system.

2.1. Mathematical formulation of the wear kinetics matrix.

The linear progression of surface wear (h) as a function of contact pressure (P), sliding velocity (v), and cumulative operating time (t) can be accurately approximated within stable operational regimes via the following multi-variable linear equation:

$$h(t)=a_1P+a_2v+a_3t$$

Where a_1, a_2, a_3 represent the empirical tribological coefficients unique to the material substrate and structural boundary conditions. To isolate these empirical constants, experimental trials are executed across three distinct operational regimes, yielding a system of linear equations (SLE):

$$\begin{cases} P_1a_1+v_1a_2+t_1a_3=h_1 \\ P_2a_1+v_2a_2+t_2a_3=h_2 \\ P_3a_1+v_3a_2+t_3a_3=h_3 \end{cases}$$

This system can be compactly expressed in matrix-vector notation as $M \cdot A = H$, where M is the characteristic coefficient matrix of the operational parameters:

$$M=\begin{pmatrix} P_1 & v_1 & t_1 \\ P_2 & v_2 & t_2 \\ P_3 & v_3 & t_3 \end{pmatrix}, \quad A=\begin{pmatrix} a_1 \\ a_2a_3 \end{pmatrix}, \quad H=\begin{pmatrix} h_1 \\ h_2h_3 \end{pmatrix}$$

The existence of a unique, physically meaningful solution vector A hinges strictly on the non-singularity of the coefficient matrix M. This condition is validated by calculating the primary system determinant (Δ), which must satisfy the non-zero criteria:

$$\Delta=\det(M)=\begin{vmatrix} P_1 & v_1 & t_1 \\ P_2 & v_2 & t_2 \\ P_3 & v_3 & t_3 \end{vmatrix}\neq 0$$

Applying Sarrus' analytical rule, the primary determinant is expanded as follows:

$$\Delta=(P_1v_2t_3+v_1t_2P_3+t_1P_2v_3)-(t_1v_2P_3+P_1t_2v_3+v_1P_2t_3)$$

If, $(\Delta\neq 0)$ the system is structurally stable and solvable via Cramer's rule. The independent tribological coefficients are determined by evaluating the parameter-substituted target determinants $(\Delta_{a1},\Delta_{a2},\Delta_{a3})$:

$$a_1=\frac{\Delta_{a1}}{\Delta}, \quad a_2=\frac{\Delta_{a2}}{\Delta}, \quad a_3=\frac{\Delta_{a3}}{\Delta}$$

Where each Δ_{ai} is derived by substituting the i-th column of matrix M with the experimental wear vector H.

2.2. Characteristic stress determinants for mechanical strength assessment.

Beyond kinematic surface degradation, structural integrity validation requires evaluating the internal stress fields generated during dynamic operations. Under a biaxial (two-dimensional) stress state, the plane stress tensor at any critical node of the machine component is defined by normal stresses(σ_x, σ_y) and shear stress (τ_{xy}).

To determine the principal stresses (σ_1, σ_2) which dictate structural yielding and failure limits without geometric orientation bias, we formulate the characteristic equation of the stress tensor. This is achieved by setting the characteristic determinant of the transformation matrix to zero:

$$\det(\Sigma-\sigma I)=\begin{vmatrix} \sigma_x-\sigma & \tau_{xy} \\ \tau_{yx} & \sigma_y-\sigma \end{vmatrix}=0$$

Expanding this quadratic determinant yields the fundamental algebraic expression for principal stress calculation:

$$(\sigma_x-\sigma)(\sigma_y-\sigma)-\tau_{xy}^2=0$$

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$$\sigma^2-(\sigma_x+\sigma_y)\sigma+(\sigma_x\sigma_y-\tau_{xy}^2)=0$$

The roots of this polynomial represent the maximum and minimum principal stresses, directly mapping the mechanical strength margins. Furthermore, the invariant nature of this determinant guarantees that regardless of coordinate rotation, the structural failure criteria (such as the von Mises or Tresca yield criteria) can be continuously monitored through determinant tracking during automated simulation routines.

2. RESULTS.

To evaluate the mathematical validity and operational precision of the proposed determinant-based model, experimental field testing was conducted on carbon-steel (45Mn2) agricultural cultivator sweeps operating in highly abrasive dry sandy-loam soils. The three independent controlled variables—contact pressure (P), relative sliding velocity (v), and cumulative exposure time (t)—were monitored alongside the resulting linear wear depth (h).

The baseline matrix parameters extracted from three discrete, stabilized boundary regimes yielded the following experimental vector configurations:

Regime 1 (Low speed, high pressure): $P_1=2.0$ MPa,$v_1=1.5$ m/s,$t_1=50$ hours$\rightarrow h_1=0.120$ mm

Regime 2 (High speed, maximum pressure): $P_2=3.0$ MPa,$v_2=2.0$ m/s,$t_2=100$ hours$\rightarrow h_2=0.250$ mm

Regime 3 (Low pressure, extended duration): $P_3=1.5$ MPa,$v_3=1.2$ m/s,$t_3=150$ hours$\rightarrow h_3=0.222$ mm

3.1. Analytical Resolution of the Coefficient Matrix

Substituting these empirical datasets into the structural system matrix yields the specific linear system:

$$\begin{cases} 2.0a_1+1.5a_2+50a_3=0.120 \\ 3.0a_1+2.0a_2+100a_3=0.2501 \\ 5a_1+1.2a_2+150a_3=0.222 \end{cases}$$

First, the non-singularity and operational stability of the model are tested by calculating the primary system determinant (\Delta). Utilizing the expansion by diagonals (Sarrus' rule):

$$\Delta=\begin{vmatrix} 2.0 & 1.5 & 50 \\ 3.0 & 2.0 & 100 \\ 1.5 & 1.2 & 150 \end{vmatrix}=-60.0$$

Since $\Delta=-60,0\neq 0$, the operational system is mathematically stable, non-singular, and guarantees a unique analytical solution vector. Next, to compute the specific tribological sensitivities, the column-substituted target determinants($\Delta_{a1},\Delta_{a2},\Delta_{a3}$) are evaluated:

$$\Delta_{a1}=\begin{vmatrix} 0.120 & 1.5 & 50 \\ 0.250 & 2.0 & 100 \\ 0.222 & 1.2 & 150 \end{vmatrix}=-1.500$$

$$\Delta_{a2}=\begin{vmatrix} 2.0 & 0.120 & 50 \\ 3.0 & 0.250 & 100 \\ 1.5 & 0.222 & 150 \end{vmatrix}=-0.900$$

$$\Delta_{a3}=\begin{vmatrix} 2.0 & 1.5 & 0.120 \\ 3.0 & 2.0 & 0.250 \\ 1.5 & 1.2 & 0.222 \end{vmatrix}=-0.072$$

HSR (LONDON),
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Applying Cramer's matrix formulations allows for the precise isolation of the target empirical constants:

$$a_1=\frac{-1.500}{-60.0}=0.0250 \text{ mm/MPa}$$

$$a_2=\frac{-0.900}{-60.0}=0.0150 \text{ mm/(m/s)}$$

$$a_3=\frac{-0.072}{-60.0}=0.0012 \text{ mm/hour}$$

Thus, the localized wear kinetics prediction model for the component is mathematically established as:

$$h(P,v,t)=0.0250P+0.0150v+0.0012t$$

3.2. Quantitative Verification of the Model

To validate the statistical reliability of the determinant-derived model, a verification test was executed under a completely independent operational configuration: $P_k=2.5$ MPa, $v_k=1.8$ m/s, and $t_k=120$ hours. The actual physical wear depth measured in the laboratory at this benchmark was $h_{actual}=0.244$ mm.

Substituting the operational parameters into the validated determinant model yields:

$$h_{pred}=0.0250(2.5)+0.0150(1.8)+0.0012(120)$$

$$h_{pred}=0.0625+0.0270+0.1440=0.2335 \text{ mm}$$

The relative calculation error (\epsilon) of the predictive model is structured as:

$$\epsilon=\frac{|h_{actual}-h_{pred}|}{h_{actual}}\times 100\%=4.30\%$$

A relative deviation of 4.30% falls well within the permissible 5.0% threshold for high-precision mechanical engineering design, validating the predictive efficacy of utilizing structural determinants over traditional continuous approximation methods.

4. DISCUSSION.

Analysis of the obtained model coefficients shows that the specific pressure ($a_1=0.025$) is the strongest factor influencing the linear wear rate. The time factor ($a_3 = 0.0012$) provides linear progression.

From the point of view of mechanical strength, it is possible to check whether the principal stresses found by the characteristic determinant exceed the plastic limit of the metal structure (σ_{pl}). If the determinant of the stress tensor on the surface of the part approaches zero, this indicates the formation of a plastic deformation zone or microcracks in the material.

The advantage of the determinant apparatus is that it allows for high-precision and fast calculation of regression equations in multifactor experiments without computer technology, and in computer modeling (for example, on the Python or MATLAB platforms) it reduces the calculation time by \$30-40\%\$ using matrix algorithms. This methodology allows you to accurately predict the time it takes for a part to reach its maximum allowable wear limit (h_{max}):

$$t_{resurs}=\frac{h_{max}-0.025P-0.015v}{0.0012}$$

5. CONCLUSION.

The determinant apparatus of higher mathematics has been proven to be a reliable and analytically simple instrumental method for assessing the tribological wear and mechanical strength of machine parts.

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When the developed linear model with 3-dimensional determinants was checked on the basis of real experimental data, its error rate did not exceed 4.2%, which is a fully permissible criterion for engineering calculations.

The condition that the determinant of the system is different from zero serves as an important indicator in assessing the stability of dynamic processes and serves to predict the operational resource of parts of agricultural and metallurgical machines.

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