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DYNAMIC MATHEMATICAL MODELING OF LOAD MASS CHANGES DURING TRAILER LIFTING AND TIPPING PROCESSES

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Abstract: This paper develops a mathematical model of the change in load mass during the operation of a trailer lifting-tipping device. Five characteristic positions of the load in the body during unloading are considered. Calculated dependencies are obtained for each position and combined into a system of equations with boundary conditions.

Keywords: Trailer, lifting-tipping device, hydraulic system, load position diagrams, unloading process, mathematical modeling.

1. Introduction

The operation of a trailer's lifting-tipping device (LTD) is a complex, multi-factor problem. During unloading, the load mass decreases continuously and unevenly — down to complete emptying of the body. At the same time, the resistance force acting on the lift ram decreases, which leads to its acceleration and the emergence of dynamic loads in the trailer structure. Accounting for these loads is of fundamental importance when designing vehicles of this class.

An analysis of domestic and foreign technical sources, dissertations, and internet resources [1–5] shows that this problem has not yet received a comprehensive scientific solution. In particular, there is no systematized mathematical model describing the change in load mass during unloading for various types of bulk and viscous materials.

The purpose of this work is to build a mathematical model of the change in load mass during the operation of a trailer's LTD for various discharge mechanisms, and, on this basis, to form a system of equations with boundary conditions suitable for engineering calculations of the device's design parameters.

Depending on the type of load (bulk or viscous) and its moisture content (dry or wet), unloading from the trailer body is carried out by one of three mechanisms:

Mutual sliding of load particles (dry bulk loads: grain, dry sand, and similar easily poured materials). The particles slide and partially roll over each other's surfaces, imitating the flow of a liquid. The process begins immediately after the side is opened and ends before the tipping angle reaches the angle of friction.

Sliding of the load along the bearing surface of the platform (cotton, stalks of various crops, stone, brick, and other hard-to-pour loads). The load shifts predominantly as a single whole along the bottom of the body; partial mutual sliding of particles is also present. With single or portioned discharge, significant dynamic loads arise.

Mixed discharge mechanism (gravel, wet sand, coal, soil, and similar materials with intermediate properties). Initially, the upper part of the load is discharged in portions through mutual particle sliding, after which the lower part shifts as a single block along the body surface. Dynamic loads in this case are moderate.

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The model is based on the principle of geometric similarity: during unloading, the load retains the shape of the body, and its mass changes solely due to a decrease in the area of its vertical cross-section. The mass of the load in the body is determined by the mechanics formula:

$$\mathbf{m_1 = \rho \cdot V,} \quad (1)$$

where ρ is the density of the load, kg/m³; V is the volume of the load in the body, m³.

Observations show that during unloading, the volume of the load changes primarily due to a change in its vertical cross-sectional area. Denoting this cross-sectional area as S, and the length of the body as B, we obtain:

$$\mathbf{V = S \cdot B,} \quad (2)$$

Substituting (2) into (1):

$$\mathbf{m_1 = \rho \cdot B \cdot S.} \quad (3)$$

Since ρ and B remain constant during unloading, the change in load mass is entirely determined by the change in the area S of its vertical cross-section.

Characteristic load positions and cross-sectional areas

Depending on the tipping angle α of the platform, the vertical cross-section of the load takes one of five characteristic shapes. Let us introduce the following notation: φ_0 — the angle of natural repose of the load during free loading; γ — the angle of artificial repose; А — the height of the load; Б — the width of the load; $\beta = 90° - \varphi_0 + \alpha$ — the angle of natural repose in the vertical plane.

Position 1. The sides are closed, the body is fully loaded, the platform is horizontal ($\alpha = 0°$; $\varphi_0 = \gamma = 90°$). The cross-section of the load is a rectangle:

$S = S_2 = А \cdot Б$; $m_1 = \rho \cdot А \cdot Б \cdot B = m_{1max}$.

Position 2. The side is open, the platform is horizontal ($\alpha = 0°$). Part of the load has spontaneously spilled out, forming a natural angle of repose $\beta = 90° - \varphi_0$. The cross-section is a one-sided trapezoid.

Position 3. The platform is tilted at angle α ($0° < \alpha \leq \arctan(А/Б)$). The cross-section is a one-sided trapezoid.

For positions 2, 3, and 4, the area of the rectangle is $S_2 = А \cdot Б$, and the area of the spilled-off triangle is:

$x = А \cdot \tan(90° - \varphi_0 + \alpha)$, $S_3 = (А^2 \cdot \tan(90° - \varphi_0 + \alpha)) / 2$.

The resulting cross-sectional area:

$$\mathbf{S = S_2 - S_3 = (2 \cdot А \cdot Б - А^2 \cdot \tan(90° - \varphi_0 + \alpha)) / 2.} \quad (4)$$

Substituting (4) into (3), we obtain the equation for the change in load mass for positions 2, 3, and 4:

$$\mathbf{m_1 = (\rho \cdot B \cdot [2 \cdot А \cdot Б - А^2 \cdot \tan(90° - \varphi_0 + \alpha)]) / 2.} \quad (I)$$

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Position 4. The natural angle of repose of the load coincides with the diagonal of the body; the cross-section is a right triangle. The calculation follows formula (I) at the corresponding value $\alpha = \arctan(А/Б)$.

Position 5. The height of the remaining load is less than the initial height ($\arctan(А/Б) < \alpha \leq \varphi_0$); the cross-section is a right triangle. The height A is determined from:

$$\mathbf{A = Б / \tan(90° - \varphi_0 + \alpha).} \quad (5)$$

The area of the triangular cross-section:

$S_3 = А \cdot Б / 2.$

Substituting (5):

$$\mathbf{S = (0.5 \cdot Б^2) / \tan(90° - \varphi_0 + \alpha).} \quad (7)$$

The mass change equation for position 5:

$$\mathbf{m_1 = (0.5 \cdot \rho \cdot В \cdot Б^2) / \tan(90° - \varphi_0 + \alpha).} \quad (II)$$

3.2. System of equations for load mass change

Combining equations (I) and (II) with boundary conditions, we obtain the complete system of equations describing the change in load mass during mutual particle sliding throughout the entire unloading cycle:

$m_1 = \rho \cdot В \cdot (2 \cdot А \cdot Б - А^2 \cdot \tan(90° - \varphi_0 + \alpha)) / 2 = m_{1max}$, at $\alpha = 0°$, $\varphi_0 = 90°$;

$m_1 = \rho \cdot В \cdot (2 \cdot А \cdot Б - А^2 \cdot \tan(90° - \varphi_0 + \alpha)) / 2$, at $\alpha = 0°$, $\varphi_0 = \varphi_0$;

$m_1 = \rho \cdot В \cdot (2 \cdot А \cdot Б - А^2 \cdot \tan(90° - \varphi_0 + \alpha)) / 2$, at $0° < \alpha \leq \arctan(А/Б)$, $\varphi_0 = \varphi_0$;

$m_1 = (0.5 \cdot \rho \cdot В \cdot Б^2) / \tan(90° - \varphi_0 + \alpha)$, at $\arctan(А/Б) < \alpha \leq \varphi_0$, $\varphi_0 = \varphi_0$;

$m_1 = 0$, at $\alpha > \varphi_0$.

When unloading loads of the second type (sliding along the bearing surface of the platform), the load moves off as a single block without changing its cross-section, so its mass remains constant right up until the moment of sudden discharge. This corresponds to the stepwise nature of the mass change on the diagram (Fig. 6 of the source).

4. Discussion

The dependencies obtained allow several practically important observations to be made.

The load mass changes inversely with the tipping angle: the greater the angle, the smaller the residual mass of the load in the body. For mutual particle sliding (the first type), the dependence $m_1(\alpha)$ is nonlinear: in the range $0° < \alpha \leq \arctan(А/Б)$, the mass decreases along a curve (with acceleration), while in the range $\arctan(А/Б) < \alpha \leq \varphi_0$, it decreases linearly (a sharp decrease).

Boundary regimes deserve special attention. If $\varphi_0 = \varphi$ (the angle of natural repose equals the angle of friction against the platform surface), discharge occurs primarily due to the internal friction angle of the particles; the bottom layer, in contact with the platform, discharges due to the external

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friction angle (tan φ = f). If $\varphi_0 > \varphi$, then upon reaching angle $\alpha = \varphi$, the entire remaining load in the body discharges instantaneously, accompanied by a shock dynamic impact on the structure.

When unloading loads of the second type (sliding along the platform), sudden simultaneous discharge occurs at $\alpha = \varphi$ — the angle of friction of the load against the platform. This moment is critical from the standpoint of dynamic loads: the abrupt vanishing of the resistance force causes a sudden increase in the speed of the lift ram.

Thus, an uncontrolled change in the tipping angle near the value equal to the friction angle creates conditions for the emergence of significant dynamic forces, which negatively affect the service life of the trailer structure and frame.

The proposed mathematical model can be used in the design of LTDs for calculating instantaneous load values on the hydraulic cylinder ram of the lifting mechanism, optimizing the kinematics of the mechanism, and developing hydraulic system control algorithms to ensure smooth unloading.

5. Conclusion

Based on the results of the study, the following conclusions are formulated:

The mass of the load in the body changes inversely with the tipping angle of the platform.

For mutual particle sliding (first type): if $\varphi_0 = \varphi$, discharge occurs due to the internal friction angle (the bottom layer discharges due to the external friction angle, tan φ = f); if $\varphi_0 > \varphi$, the remaining load discharges simultaneously at $\alpha = \varphi$, creating a shock load.

For sliding along the platform (second type), the load also discharges simultaneously at the angle α equal to the friction angle against the platform surface.

In the range $0° < \alpha \leq \arctan(А/Б)$, the load mass decreases along a curvilinear dependence with a uniformly accelerated character of change; in the range $\arctan(А/Б) < \alpha \leq \varphi_0$, it decreases linearly with a sharp drop.

The passage of the tipping angle through a value equal to the sliding friction angle is accompanied by an uneven change in load, which provokes dynamic forces and has a negative effect on the trailer structure.

The developed mathematical model of load mass change during unloading from a trailer has important practical significance for calculating the main parameters of the lifting-tipping device and ensuring reliable operation of its hydraulic system.

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