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THE CAYLEY–HAMILTON METHOD FOR FINDING THE INVERSE OF SECOND- AND THIRD-ORDER SQUARE MATRICES

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Abstract. This paper presents a method for finding the inverse matrix based on the Cayley–Hamilton theorem. Using the characteristic polynomial of a matrix, the method makes it possible to express the inverse matrix in terms of powers of the matrix.

Keywords: inverse matrix, characteristic polynomial, Cayley–Hamilton theorem, square matrix.

There are several methods for finding the inverse matrix in linear algebra: the method of algebraic complements, the Gauss–Jordan method of elementary transformations, and the Cayley–Hamilton method. The last of these makes it possible to obtain the inverse matrix using only the operations of matrix multiplication and addition.

Theorem (Cayley–Hamilton). Every square matrix satisfies its own characteristic equation. If an n-th order matrix A has the characteristic polynomial

$$\chi(\lambda) = \det(\lambda E - A) = \lambda^n + a_{n-1}\lambda^{n-1} + \ldots + a_1\lambda + a_0$$

then

$$A^n + a_{n-1}A^{n-1} + \ldots + a_1 A + a_0 E = 0$$

Essence of the method. In this equality $a_0 = (-1)^n \det A$. . If $\det A \quad 0$, then $a_0 \quad 0$ holds. Multiplying both sides of the equality by A^{-1}, we isolate $a_0 A^{-1}$:

$$A^{-1} = -\frac{1}{a_0}\left(A^{n-1} + a_{n-1}A^{n-2} + \ldots + a_2 A + a_1 E\right)$$

Algorithm:

1) Form the characteristic polynomial $\chi(\lambda) = det(\lambda E - A)$;

2) Check the condition $a_0 \quad 0$ (this means that the matrix A is non-singular);

3) Compute the powers $A, A^2, \ldots, A^{n-1}$;

4) Substitute the values found into the formula above to obtain A^{-1}.

Example. Let the matrix $A = \begin{matrix} 1 & 2 \\ 3 & 4 \end{matrix}$ be given. Then $\chi(\lambda) = \lambda \mathrm{I} - 5\lambda - 2$, i.e. $a_1 = -5, a_0 = -2$. By the formula,

$$A^{-1} = -\frac{1}{2}(A - 5E) = \frac{1}{2}\begin{matrix} -4 & 2 \\ 3 & -1 \end{matrix} = \begin{matrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{matrix}$$

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Verification: $A\cdot A^{-1}=\begin{matrix}1&2\\3&4\end{matrix}\begin{matrix}-2&1\\\frac{3}{2}&-\frac{1}{2}\end{matrix}=\begin{matrix}-2+3&1-1\\-6+6&3-2\end{matrix}=\begin{matrix}1&0\\0&1\end{matrix}=E$

Example. Given the matrix $A=\begin{matrix}3&1&3\\1&-1&0\\0&1&1\end{matrix}$. Let us find its inverse. Here $\chi(\lambda)=\lambda^3-3\lambda^2-2\lambda+1$, i.e. $a_2=-3, a_1=-2, a_0=1$. Now, using the formula, we compute the inverse: $A^{-1}=\frac{1}{4}(A^2-3A-2E)$. By the Cayley–Hamilton theorem, $A^3-3A^2-2A+E=0$. Hence, the inverse matrix formula is:

$$A^{-1}=-A^2+3A+2E=-\begin{matrix}-10&-5&-12\\-2&-2&-3\\-1&0&-1\end{matrix}+\begin{matrix}9&3&9\\3&-3&0\\0&3&3\end{matrix}+\begin{matrix}2&0&0\\0&2&0\\0&0&2\end{matrix}=\begin{matrix}1&-2&-3\\1&-3&-3\\-1&3&4\end{matrix}$$

Verification:

$$A\cdot A^{-1}=\begin{matrix}3&1&3\\1&-1&0\\0&1&1\end{matrix}\begin{matrix}1&-2&-3\\1&-3&-3\\-1&3&4\end{matrix}=\begin{matrix}3+1-3&-6-3+9&-9-3+12\\1-1+0&-2+3+0&-3+3+0\\0+1-1&0-3+3&0-3+4\end{matrix}=\begin{matrix}1&0&0\\0&1&0\\0&0&1\end{matrix}=$$

Conclusion. The Cayley–Hamilton method gives a single formula for finding the inverse matrix and is convenient to program. It is efficient for matrices of small order $(n\ \ 3,4)$. For matrices of large order, however, computing the characteristic polynomial becomes difficult, so the Gauss–Jordan method is preferable.

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