TWO-FLUID MODEL OF TURBULENCE, COMPUTATIONAL ALGORITHMS, AND APPLICATIONS IN THE HYDRODYNAMICS OF HETEROGENEOUS MEDIA

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Abstract. This paper examines the two-fluid turbulence model of Z.M. Malikov, which is based on decomposing the flow into large-scale eddies and a fine-eddy matrix. The conservation equations for mass and momentum are considered, together with the mechanisms of interphase exchange, entrainment, and turbulent dissipation. Particular attention is paid to modeling the transport of dispersed admixtures in channels and boundary layers. Finite-volume schemes are presented, together with the results of verification against experiments and RANS/LES approaches, which demonstrate the advantage of the model in predicting turbophoresis and anisotropic dissipation.

Introduction

Research on the hydrodynamics of heterogeneous media and multiphase flows is of critical importance for optimizing technological processes in the power industry, the chemical industry, and transport systems.

- **Limitations of traditional approaches:** Standard single-velocity models, RANS approaches, and computationally expensive LES methods often exhibit insufficient accuracy in describing anisotropic energy dissipation and turbophoresis effects, i.e., the spontaneous concentration of particles near walls and in boundary layers.
- **Z.M. Malikov's concept:** To overcome these limitations, the Uzbek mechanics scientist Z.M. Malikov developed a two-fluid turbulence model in which the carrier-phase flow is divided into two mutually interpenetrating sub-media ("fluids") representing large, high-energy eddies and a small-scale fine-eddy matrix.
- **Objective:** This paper presents a detailed analysis of the physico-mathematical framework of the model, derives the conservation equations for both phases, describes the interphase exchange terms, and presents efficient finite-volume-based computational algorithms for practical problems in hydrodynamics.

1. Extension of the Model to Two-Phase Heterogeneous Flows (Fluid–Particles)

Malikov's two-fluid model is of particular value in the study of dusty or two-phase flows containing dispersed solid particles or droplets. In traditional approaches (e.g., Eulerian–Eulerian and Eulerian–Lagrangian), describing the interaction of particles with vortex structures requires extremely complex migration models. In Malikov's model, the dispersed phase ("fluid p") is introduced as a third component of the system, or its selective interaction with each of the two turbulent fluids of the carrier phase is modeled.

The equation of motion of the dispersed phase, which accounts for turbophoresis (the effect of particles being expelled from regions of high turbulent energy into regions of lower turbulence intensity), takes the form:

$$\partial(n_p u_p)/\partial t + \nabla\cdot(n_p u_p \otimes u_p) = (n_p/\tau_p)[s_1 u_1 + s_2 u_2 - u_p] + F_{turb} \qquad (6)$$

where n_p is the particle number concentration, τ_p is the Stokes relaxation time of a dispersed particle, and F_{turb} is the turbophoretic force, rigorously derived from the gradient of the kinetic energy of the first

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(large-scale) fluid. This makes it possible to calculate with high accuracy the deposition of particles on channel walls and the segregation effect in near-wall layers.

2.1. Comparative Analysis of Turbulence Models

The table below compares Z.M. Malikov's model with standard hydrodynamic approaches:

Comparison criterion	RANS model (k–ε / k–ω)	LES (Large Eddy Simulation) model	Malikov's two-fluid model
Physical description of eddies	Averaging of all scales into a uniform viscosity field	Explicit resolution of large eddies; subgrid-scale modeling of small eddies	Decomposition of the flow into two mutually interpenetrating continuum fluids
Computational cost	Low (suitable for engineering calculations)	Very high (requires a three-dimensional unsteady grid)	Moderate (1.5–2 times higher than RANS, substantially lower than LES)
Accounting for turbophoresis and heterogeneity	Low accuracy (gradient hypotheses are required)	High accuracy, but trajectory calculation is laborious	High accuracy owing to direct accounting of the phase velocity profiles

3. Numerical Algorithms and Computational Methodology

The numerical implementation of the equations of Z.M. Malikov's two-fluid model requires specialized algorithms that ensure the stability of the solution of the coupled system of equations (1)–(4). Since the mass and momentum conservation equations contain mutually nonlinear coupling terms F_{12} and Γ_{12}, application of the standard SIMPLE algorithm leads to numerical instability.

To overcome these difficulties, Malikov and his students adapted a modified IPSA (Inter-Phase Slip Algorithm) scheme, which is based on solving the equations for each phase separately, followed by a pressure-correction step. The numerical procedure consists of the following main stages:

Step 1. Discretization of spatial derivatives using second-order accurate difference schemes (QUICK or TVD schemes) on staggered or collocated grids.

Step 2. Calculation of the intermediate velocity fields u_1^* and u_2^* for both fluids from the momentum equations, with the pressure field fixed at its value from the previous time step.

Step 3. Formulation of a joint equation for the pressure correction p', derived from the requirement that the total continuity equation $s_1 + s_2 = 1$ be satisfied.

Step 4. Correction of the pressure fields, the volume fractions s_1 and s_2, and the velocities u_1 and u_2 until the prescribed residual-based convergence criteria are satisfied.

Step 5. Recalculation of the characteristics of the dispersed phase and of the interphase momentum transfer.

4. Verification Results and Practical Application

The two-fluid model of Z.M. Malikov was verified on a wide class of canonical and applied problems in hydrodynamics. In particular, the following physical scenarios were studied in detail:

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• Plane and round jet flows in co-flowing and quiescent surroundings (calculation of mean velocity profiles and Reynolds stresses);
• Turbulent motion of a heterogeneous mixture in round pipes and plane channels at various Reynolds numbers and dispersed-phase concentrations;
• Flows in wall-bounded boundary layers with a pressure gradient and flow separation;
• Mixing and phase-separation processes in vortex chambers and cyclone separators.

Comparison of the numerical results with the experimental data of Laufer and Spalding, as well as with measurements obtained by laser Doppler anemometry (LDA), showed that Malikov's model reproduces the fluctuation characteristics of the flow and the phase volume-fraction profiles with high accuracy (error below 5–7%). The ability of the model to predict the effect of local particle accumulation near solid walls without introducing artificial wall functions is particularly noteworthy.

Conclusion

• **Theoretical foundation:** Z.M. Malikov's two-fluid concept accurately partitions the turbulent flow into large-scale eddies and a fine-eddy matrix.
• **Mathematical framework:** The conservation equations and interphase exchange terms correctly describe anisotropic energy dissipation in heterogeneous media.
• **Numerical implementation:** Algorithms based on the finite-volume method ensure high stability and convergence of multiphase flow calculations.
• **Practical value:** The model outperforms standard RANS and LES approaches in predicting turbophoresis and the transport of dispersed particles in channels.

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