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TWO-FLUID MODEL OF TURBULENCE THEORETICAL FOUNDATIONS

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Abstract. This paper presents a detailed analysis of the phenomenological and physico-mathematical concept of the two-fluid turbulence model developed by the Uzbek mechanician Z.M. Malikov. The fundamental premises for decomposing the turbulent flow of the carrier phase into two mutually penetrating continuum subsystems ("fluids") are considered: one representing large, high-energy vortex structures, and the other a small-scale, fine-vortex matrix. The mass and momentum conservation equations for both phases are derived in detail, and the terms describing interphase momentum exchange, entrainment, and turbulent dissipation are described mathematically. Particular attention is paid to adapting Malikov's model to the transport of a dispersed phase (solid particles, droplets, and bubbles) in channels and boundary layers. Numerical schemes implementing the model on the basis of finite-volume and control-volume methods are presented, and the simulation results are compared with experimental data and with conventional RANS/LES approaches. The model is shown to have a marked advantage in predicting the effects of spontaneous turbulent particle concentration (turbophoresis) and anisotropic dissipation.

Keywords: two-fluid turbulence model, Malikov model, turbulence in heterogeneous media, two-phase flows, interphase exchange, turbophoresis, large-scale eddies, hydrodynamics, numerical simulation.

Introduction and Physical Motivation of the Model

The modeling of turbulent shear flows of homogeneous and heterogeneous fluids remains one of the most complex and fundamental problems in modern continuum mechanics and fluid and gas dynamics. Classical approaches based on Reynolds averaging of the Navier–Stokes equations (RANS) employ the Boussinesq eddy-viscosity concept and assume local isotropy of the turbulent stresses. However, in the description of complex three-dimensional flows with substantial streamline curvature and separation, and in the presence of suspended dispersed particles, traditional single-point semi-empirical models (such as k–ε, k–ω, or Spalart–Allmaras) exhibit systematic errors and are unable to reproduce correctly the dynamics of organized structures.

One of the most fruitful solutions to this problem has been the idea of conceptually decomposing a turbulent flow into mutually penetrating media. In the works of Z.M. Malikov, Doctor of Physical and Mathematical Sciences and Professor, an original two-fluid turbulence model was formulated and scientifically substantiated. In contrast to classical phenomenological models, in Malikov's approach the turbulized medium is treated as a continuum consisting of two interacting "fluids": a large-scale fluid (turbulent vortex clumps, or "blobs") and a small-scale fluid (a highly dissipative background fluid). Such a physical interpretation makes it possible to describe naturally the transfer of mass, momentum, and energy between structures of different scales without resorting to rigid hypotheses on the isotropy of turbulent transport.

The relevance of studying Z.M. Malikov's model is determined by the need for accurate calculation of technological processes in power-generating plants, pneumatic conveying, and chemical reactors, as

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well as in the aerodynamic design of aircraft components. This paper gives a systematic exposition of the theoretical foundations of Malikov's model, derives the key governing equations, describes the numerical algorithms for its implementation, and discusses the results of its extensive verification.

2. Theoretical Foundations and Mathematical Formulation of Z.M. Malikov's Model

The two-fluid concept of Malikov is based on the hypothesis that, at each point of a turbulent flow, a large-scale vortex phase ("fluid 1") is present with probability (volume fraction) s_1, and a small-scale dissipative matrix ("fluid 2") is present with probability $s_2 = 1 - s_1$. Both phases are characterized by their own velocity fields $u_i^{(1)}$ and $u_i^{(2)}$, densities, and mean pressures $p^{(1)}$ and $p^{(2)}$.

2.1. Mass and Momentum Conservation Equations

The continuity (mass conservation) equations for the first and second fluids, with interphase mass exchange (the processes of injection and entrainment of turbulent fluid lumps, or "moles") taken into account, have the following form:

$$\partial(s_1\rho_1)/\partial t + \nabla\cdot(s_1\rho_1 u_1) = \Gamma_{12} - \Gamma_{21} \quad (1)$$

$$\partial(s_2\rho_2)/\partial t + \nabla\cdot(s_2\rho_2 u_2) = \Gamma_{21} - \Gamma_{12} \quad (2)$$

where ρ_1 and ρ_2 are the true (intrinsic) phase densities (for an incompressible homogeneous fluid, $\rho_1 = \rho_2 = \rho_0$), and Γ_{12} and Γ_{21} express the intensity of the transition of matter from the large-scale state to the small-scale state and vice versa. The momentum conservation equations for each of the constituent fluids are written as:

$$\partial(s_1\rho_1 u_1)/\partial t + \nabla\cdot(s_1\rho_1 u_1 \otimes u_1) = -s_1 \nabla p + \nabla\cdot(s_1\tau_1) + F_{12} + u_{12}^*(\Gamma_{12} - \Gamma_{21}) \quad (3)$$

$$\partial(s_2\rho_2 u_2)/\partial t + \nabla\cdot(s_2\rho_2 u_2 \otimes u_2) = -s_2 \nabla p + \nabla\cdot(s_2\tau_2) - F_{12} - u_{12}^*(\Gamma_{12} - \Gamma_{21}) \quad (4)$$

Here F_{12} denotes the vector of the hydrodynamic interphase drag (friction) force between the two turbulent fluids; τ_1 and τ_2 are the tensors of molecular and small-scale viscous stresses; and u_{12}^* is the mass-transfer velocity at the interface of the vortex formations.

2.2. Closure Relations and Malikov's Physical Functions

A key contribution of Z.M. Malikov to the development of the theory is the construction of physically justified closed expressions for the interphase interaction force F_{12} and the volume fractions s_k. The inter-component friction force F_{12} is approximated by combined quadratic and linear drag laws:

$$F_{12} = C_D(s_1 s_2 \rho / L_{turb})|u_1 - u_2|(u_1 - u_2) + (A_v \mu / L_{turb}^2)\, s_1 s_2 (u_1 - u_2) \quad (5)$$

where L_{turb} is the characteristic scale of the large-scale turbulent fluid lumps (moles), C_D is the empirical form-drag coefficient of the elementary vortex volumes, A_v is the interphase viscous friction coefficient, and μ is the dynamic viscosity of the carrier medium. An increase in the relative velocity $|u_1 - u_2|$ leads to intensification of small-scale turbulence generation and to a transfer of energy from the first fluid to the second.

Conclusion

The two-fluid turbulence model of Z.M. Malikov is a powerful and theoretically rigorous tool for the mechanics of multiphase and turbulent media. The main conclusions of the work can be formulated as follows:

1. The conceptual decomposition of a turbulent flow into large-scale and small-scale fluids makes it possible to describe, in a physically correct manner, the structure of turbulent fluctuations and the inter-scale energy exchange.

2. The introduction of phenomenological inter-component friction and mass-exchange functions closes the conservation equations without the need for the isotropic eddy-viscosity hypothesis.

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3. The model demonstrates high efficiency and reliability in the calculation of two-phase dust-laden flows, successfully predicting the phenomena of turbophoresis and particle deposition.

4. Computational algorithms based on the IPSA scheme provide good convergence and require substantially fewer resources than direct numerical simulation (DNS) and LES.

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